TEOREMA.

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Encontré en un libro, que el cubo de un número menos su raiz, es un multiplo de 6, es decir que $x^3-x=6n$, pero no lo demostraba. Hallé la demostracion y traté de sacar algun partido de esté teorema. Nada encontré, pero en mis investigaciones di con otro teorema que he aplicado a la extracion de la raiz cubica.

El teorema es el siguiente: Todo número dividida por 6, dá un residuo igual al que dá su cubo dividido por dicho número 6, o vice versa,—todo cubo dividido por 6, da un residuo igual al que da su raiz dividida por dicho número 6. Efectivamente, todo número que no es multiple de 6, puede representarse por una de las siguientes espresiones—$x+1$, $x+2$, $x+3$, $x+4$, $x+5$, siendo x igual a cero ó un multiplo de 6.

Si x es igual 0, hallaremos que los cubos de 1, 2, 3, 4, 5 ó 1, 8, 27, 64, 125, divididos por 6, dan por residuo 1, 2, 3, 4, 5.

Si x es un multiplo de 6, tendremos $(x+1)^3=x^3+3x^2+3x+1$, expresion que dividida por 6 dá por residuo 1.

$(x+2)^3=x^3+6x^2+12x+8$, expresion que dividida por 6, nos dá un residuo de 2. Asi mismo se encuentra que $\left.\begin{array}{l}(x+3)^3\\(x+4)^3\\(x+5)^3\end{array}\right\}$ divididos por 6 dan por residuo, 3, 4, 5, y quèda demonstrado el teorema.

La extraccion de la raiz cubica es una operacion algo dificil y que exige bastantes cálculos; asi es que algunos tratados elementales de aritmetica aconsejan que para la extraccion de dichas raices se proceda por tanteo, pero el teorema arriba indicado permite reducir este tanteo á limites muy estrechos tratandose de cubos perfectos. Conociendo el número final de este cubo, desde luego sabremos en que número termina su raiz, y restando de este número el residuo que da la division del cubo por 6, podremos saber en que número hader terminar el multiplo de 6, que agregado al residuo, da la raiz.

Supongamos que se pida la raiz cubica del número 493039. Este cubo acaba en 9; por consiquiente su raiz debe acabàr en 9. El residuo de la division por 6 es uno, 1, y $9-1=8$, es decir que la raiz debe ser un multiplo de 6 que acaba en 8 mas 1. 493039 es visiblemente menor que el cubo de 80 y mayor que èl cubo de 70: entre 70 y 80, no hay mas que un multiplo de 6 que acaba en 8, que es 78 y desde luego digo sin buscar mas que la raiz es $78+1=79$.

Busquemos la raiz de 35,287,552. La raiz debe acabar en 8. Siendo el residuo de la division por 6, 4, la raiz debera ser formada por un multiplo de 6 que acabe en $4+4$. El dicho cubo es mayor que 300 y menor que 350 como se puede ver por una pequeña multiplicacion. Entre estos dos números no hay mas que un multiplo de 6 que acaba en 4. Es el número 324 y sin mas tanteos digo que la raiz es $324+4=328$.

THEOREM.

I found in a book, that the cube of a number less its root, is a multiple of 6, that is to say, that $x^3 - x = 6n$, but it was not demonstrated. I found the demonstration and tried to get some practical use out of this theorem. I found nothing, but in my investigations I found another theorem which I have applied to the extraction of the cube root. The theorem is the following: Every number divided by 6 gives a remainder equal to that which its cube gives, divided by said number 6; or vice versa, every cube divided by 6 gives a remainder equal to that which its cube root gives, divided by said number 6. Therefore, every number which is not a multiple of 6 may be represented by one of the following expressions: $x+1$, $x+2$, $x+3$, $x+4$, $x+5$, x being equal to zero or a multiple of 6. If x is equal to 0 we will find that the cubes of 1, 2, 3, 4, 5, or 1, 8, 27, 64, 125 divided by 6 give for remainders 1, 2, 3, 4, 5.

If x is a multiple of 6, we will have $(x+1)^3 = x^3 + 3x^2 + 3x + 1$, an expression that divided by 6, gives 1 for a remainder.

$(x+2)^3 = x^3 + 6x^2 + 12x + 8$, an expression that divided by 6, gives us 2 for a remainder. In the same way it is found that $\left.\begin{matrix}(x+3)^3\\(x+4)^3\\(x+5)^3\end{matrix}\right\}$ divided by 6 gives for remainders 3, 4, 5 (respectively), and the theorem is demonstrated.

The extraction of the cube root is a somewhat difficult operation, and (one) which requires much calculation; thus it is that some elementary treatises of arithmetic advise that for the extraction of said cubic roots (one) should proceed by approximation, but the theorem above indicated, allows (one) to reduce this approximation to very narrow limits when treating of perfect cubes. Knowing the final number of this cube, consequently we will know in what number its root ends, and subtracting from this number the remainder which the cube divided by 6 gives, we will be able to know in what number the multiple of 6 must end which, added to the remainder, gives the root.

Let us suppose that the cube root of 493039 is asked. This cube ends in 9, consequently its root must end in 9. The remainder of the division by 6 is 1, and $9-1=8$, that is to say that the root must be a multiple of 6 which ends in 8, plus 1. 493039 is plainly less than the cube of 80 and more than the cube of 70; between 70 and 80 there is but one multiple of 6 which ends in 8, which is 78, and consequently I say without any more search that the root is $78+1=79$.

Let us try to find the root of 35,287,552. The root must end by 8. Four, 4, being the remainder of the division by 6, the root must be formed of a multiple of 6 which ends in $4+4$. The said cube is greater than 300, and less than 350, as may be seen by a small multiplication. Between these two numbers there is but one multiple of 6 which ends in 4. It is the number 324, and without more search I say that the root is $324+4=328$.

[**NOTE.** This theorem and its translation was furnished by Dr. Halsted. EDITOR.]